DVV-003-016303 Seat No. ________

M. Sc. (Mathematics) (Sem. - III) (CBCS) Examination

May / June – 2015

Course No : 3003 : Number Theory - 1

(New Course)

Faculty Code : 003
Subject Code : 016303

Time : $2\frac{1}{2}$ Hours] [Total Marks : **70**

Instructions : (1) There are five questions.
(2) All questions are compulsory.
(3) Each question carries 14 marks.

1 Select the most appropriate answer for each of following

(a) Using Euclidean algorithm we can find ________ of two integers.

(i) lcm (ii) gcd

(iii) product (iv) sum

(b) The number of primitive roots of 59 is ________

(i) 58 (ii) 30

(iii) 29 (iv) 28

(c) If $x^2+1\equiv 0$ (mod p) has a solution then p =

(i) 59 (ii) 41

(iii) 79 (iv) 19

(d) If $x^2+1\equiv 0$ (mod p) has a no solution then p =

(i) 13 (ii) 17

(iii) 29 (iv) 67

(e) If 59 divides a^2+b^2 then 59 divides ________

(i) a only (ii) b only

(iii) a and b both (iv) neither a nor b.

(f) ________ is the consequence of Euler's theorem.

(i) Wilson's theorem (ii) Hensel Lemma

(iii) Fermat's theorem (iv) Eulidean Algorithm

(g) If a is odd then the largest integer that divides a^4-1 is ________

(i) 2 (ii) 8

(iii) 4 (iv) 16

2 Attempt any two

(a) Define the greatest common divisor of two integers. **7**

If g.c.d. of two integers a and b is then prove that

$$(\frac{a}{g}, \frac{b}{g})=1.$$

(b) Suppose a and b are positive integers then prove that **7**

(a,b)[a,b]= ab.

(c) Suppose a and b are non zero integers and m is a positive integer then prove that m[a,b] =[ma,mb]. **7**

3 All compulsory

(a) Suppose p_1, p_2,..........,p_k are the first k primes. Then prove that p_1. p_2.........p_k+1 is also a prime number. **7**

(b) Suppose p and q are distinct primes each of which divides n. Prove that pq/n **3**

(c) Use the Euclidean Algorithm to find the greatest common divisor of 1001 and 347. **4**

OR

3 All compulsory

(a) Suppose m is a positive integer and (a,m) =1. **7**

Prove that $a^{\phi(m)} \equiv 1$ (mod m).

(b) Find all solutions of $x^2 \equiv -1$ (mod 13) in the complete residue system {0, 1, 2,...,12}. **4**

(c) Determine whether the congruence equation $x^2 \equiv -1$ (mod 79) has solution or not. **3**

4 Attempt any two

(a) State and prove chinese remainder theorem. **7**

(b) Find the solutions of the following congruence equations if there is any. **7**

(i) $x^2 - 1 \equiv 0 \pmod{15}$

(ii) $x^2 + 1 \equiv 0 \pmod{125}$

(c) Suppose m is a positive integer such that $m = m_1, m_2$ **7**

$(m_1, m_2) = 1$, $(\phi(m_1), \phi(m_2)) \geq 2$. Prove that m has no primitive root. Give three positive integers of the above type which have no primitive roots.

5 Do as directed. All are compulsory and each question carries two marks.

(a) Give the statement of Wilson's theorem

(b) Write the statement of Hensel's Lemma.

(c) Find the primitive roots of 7^2 and 11^2.

(d) Find the number of positive integers relatively prime to $1001 \times 25 \times 31$.

(e) Write the statement of Divison Algorithm.

(f) Give all the positive divisors of p.q.r.s where p,q,r,s are distinct prime numbers.

(g) If p is prime number and $n \geq 1$ then what is the value of $\phi(p^n)$..